Efficient Search-Based Weighted Model Integration

Zhe Zeng, Guy Van den Broeck
University of California, Los Angeles

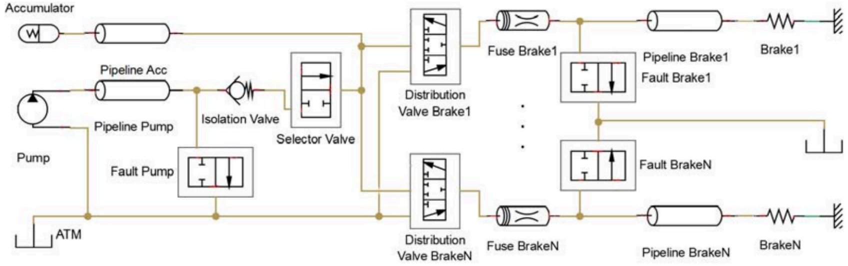
Outline

- **What is Weighted Model Integration (WMI)?**
- What structure to exploit?
- From WMI to Model Integration
- How does Search-Based MI (SMI) work?
- Complexity & Experimental Results

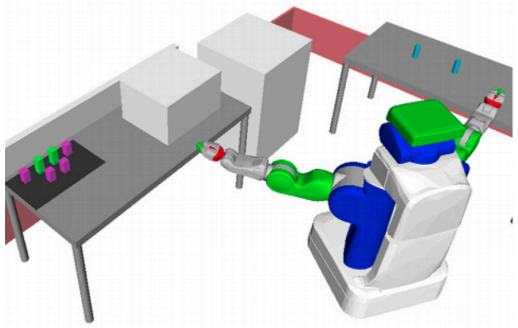
Hybrid Probabilistic Model

- Real world data are noisy, complex and heterogeneous.

Gesture-based Programming
[Figueiredo et al., 2016]



Formal Verification
[Cimatti et al., 2017]



Robotics
[Lagriffoul et al., 2018]

Hybrid Probabilistic Model

- Real world data are noisy, complex and heterogeneous.
- We need:
  - Expressive modeling languages
  - Powerful and flexible probabilistic framework:
    - hybrid models, inference, …

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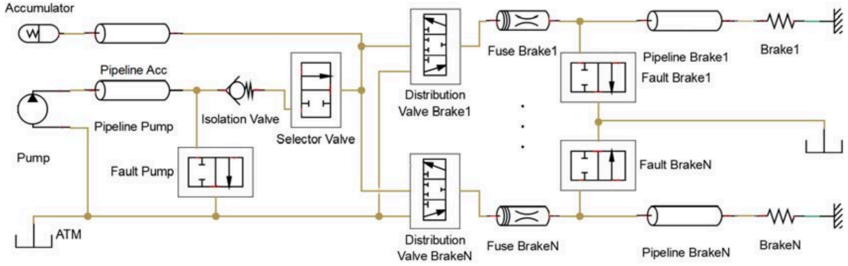

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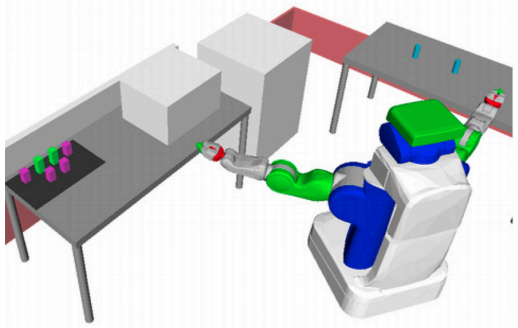

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From WMC to WMI

Domain	Discrete	Hybrid
Model Language	Propositional Formulas	SMT($\mathcal{LRA}$) Formulas
Example	$(A \lor B) \land (\neg A \lor C)$	$\begin{cases} (-1 \leq y \leq 1) \lor B \\ -0.5 \leq x_1, x_2 \leq 0.5 \\ (x_1 + 1 \leq y) \lor (y \leq x_2 - 1) \\ \neg A \lor C \end{cases}$
Probabilistic Inference	Weighted Model Counting (WMC)	Weighted Model Integration (WMI)

Weighted Model Integration (WMI)

- SMT($\mathcal{LRA}$) formulas with both propositional and $\mathcal{LRA}$-atoms
- $\mathcal{LRA}$-atoms: $\sum_i a_i x_i \bowtie b, \bowtie \in \{<, \leq, >, \geq, =, \neq\}$
- WMI definition: $WMI(\theta, w \mid \boldsymbol{x}, \boldsymbol{b}) = \sum_{\boldsymbol{b}^* \in \mathbb{B}^m} \int_{\theta(\boldsymbol{x}, \boldsymbol{b}^*)} w(\boldsymbol{x}, \boldsymbol{b}^*) d\boldsymbol{x}$ with weight w.

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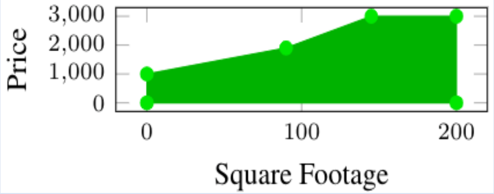
Example: House price SMT($\mathcal{LRA}$) model

Given the following SMT($\mathcal{LRA}$) model γ_i and its weight function w,

$$\gamma_i = \begin{cases} (price_i < 10 \cdot sqft_i + 1000) \ \vee \ (price_i < 20 \cdot sqft_i + 100) \\ (0 < price_i < 3000) \wedge (0 < sqft_i < 200) \end{cases}$$



$$w(\ell) = \begin{cases} price_i^2, & \ell = (0 < price_i < 3000) \\ 1, & \text{otherwise} \end{cases}$$

then its weighted model integration: $WMI(\gamma_i, w) = 8.785 \times 10^{11}$

Recap: WMI & Probabilistic Reasoning

- Probability of query q given SMT($\mathcal{LRA}$) model γ and weight functions:

$$Pr(q) = \frac{WMI(\gamma \wedge q, w)}{WMI(\gamma, w)}$$

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Example: House price model with query

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and query $q = price_i < 2000$, the probability of q can be computed as

$$Pr(q) = \frac{WMI(\gamma \wedge q, w)}{WMI(\gamma, w)} = \frac{3.926 \times 10^{11}}{8.785 \times 10^{11}} = 44.69\%$$

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Suppose that we have n houses independent from each other, and each house i has its price model as γ_i:

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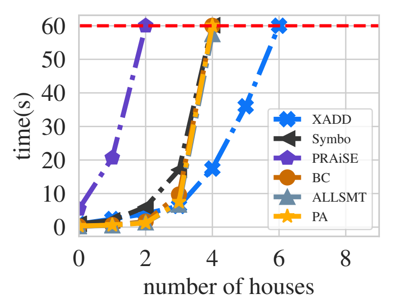
then the SMT($\mathcal{LRA}$) model of these houses is $\theta = \bigwedge_{i=1}^{n} \gamma_i$.



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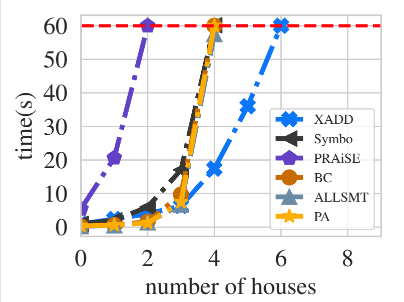

🎯 **TARGET**: to build an algorithm for WMI that exploits structure!

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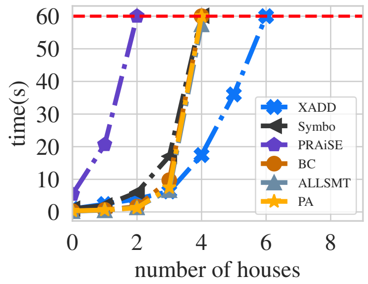

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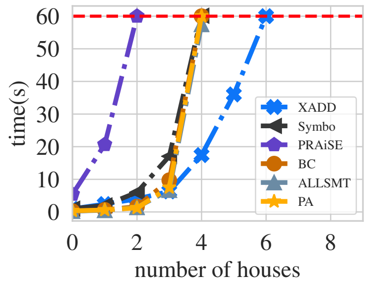

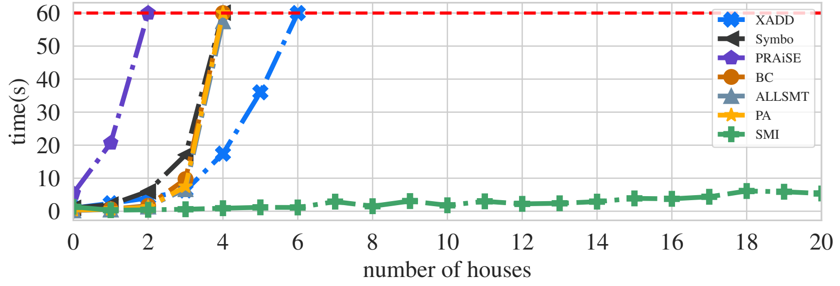

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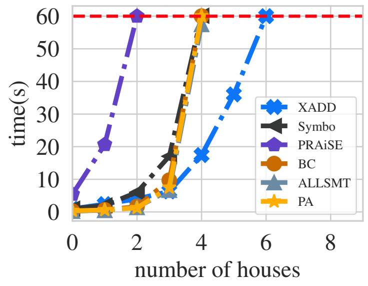

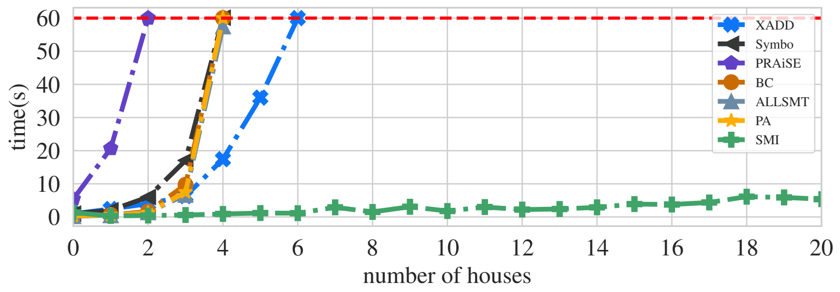

Independence helps!

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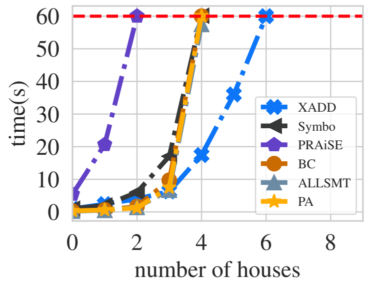

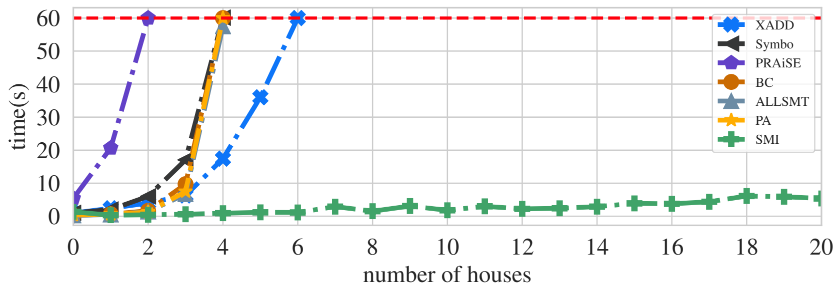

Independence helps!

- 🤔 Can we leverage conditional independence to build efficient WMI alg?

$$MI(\theta) = \int \prod_{i=1}^{n} MI(\gamma_i \mid y)dy$$

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Model Integration is all u need ☺

- Boolean to real:

$$\left[\begin{array}{ll} b & w(b) = 2 \\ \neg b & w(\neg b) = 3 \end{array}\right.$$

$$\downarrow$$

$$\left[\begin{array}{ll} \lambda_b > 0 & w(\lambda_b > 0) = 2 \\ \lambda_b < 0 & w(\lambda_b < 0) = 3 \\ -1 \leq \lambda_b \leq 1 & \end{array}\right.$$

- weighted to unweighted (monomials):

$$\left[\begin{array}{ll} x + y \geq 1 & w(x + y \geq 1) = x^2 y \\ \neg(x + y \geq 1) & w(\neg(x + y \geq 1)) = 1 \end{array}\right.$$

$$\downarrow$$

$$\left[\begin{array}{ll} x + y \geq 1 & \Rightarrow \begin{cases} \bigvee_{i=1,2} 0 \leq p_{x,i} \leq x \\ 0 \leq p_{y,1} \leq y \end{cases} \\ \neg(x + y \geq 1) & \Rightarrow \begin{cases} \bigvee_{i=1,2} 0 \leq p_{x,i} \leq 1 \\ 0 \leq p_{y,1} \leq 1 \end{cases} \end{array}\right.$$

✓ Focus on MI without loss of generality

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- In order to characterize variable dependencies
- Define Primal Graph for SMT($\mathcal{LRA}$) model:
  - Nodes – variables, Edges – between variables in the same clause

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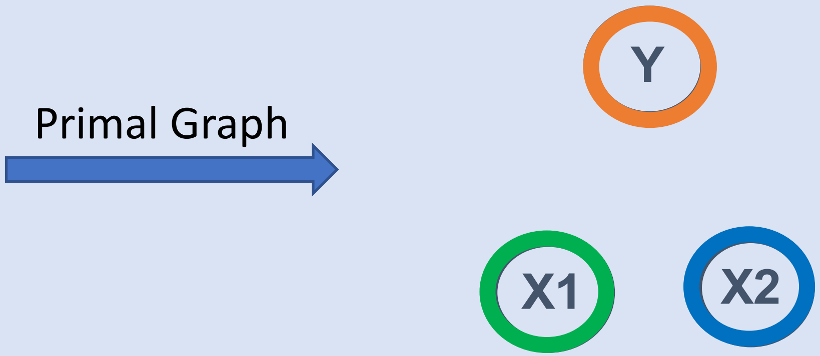

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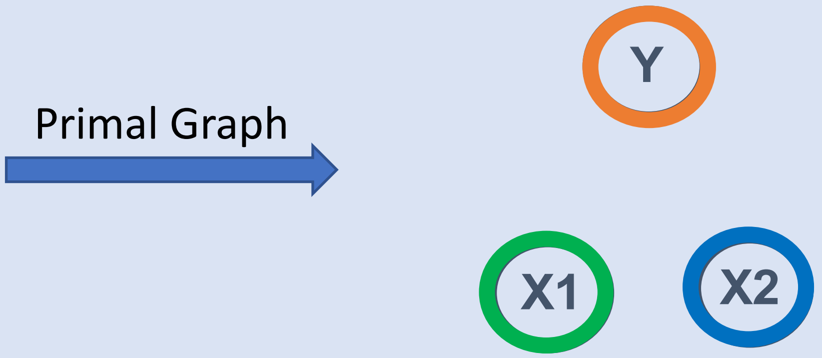

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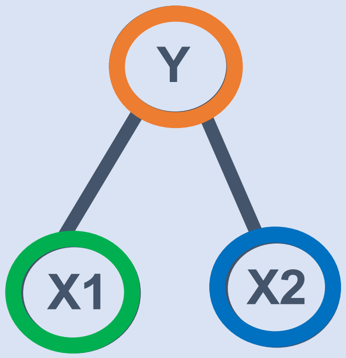

Step 2: Search guided by Primal Graph

Example: MC in discrete case

Given the following propositional formula, with *y, x1, x2* being Boolean variables,

$$(y \vee x_1) \wedge (y \vee x_2)$$

Sol: And/Or search based on primal graph

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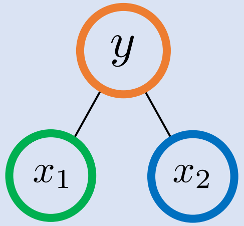
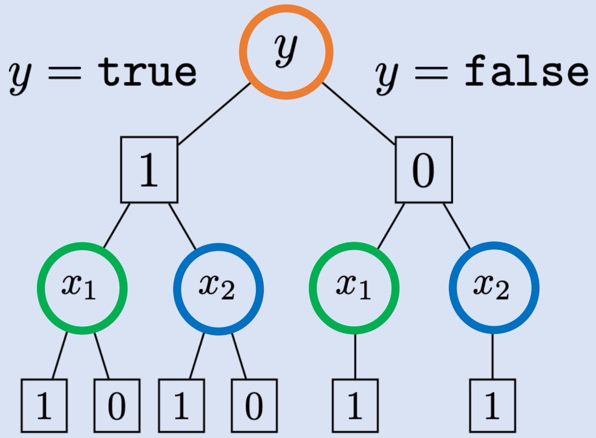


Search Tree



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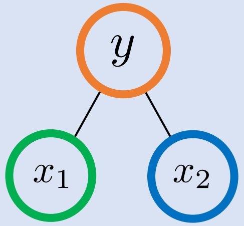

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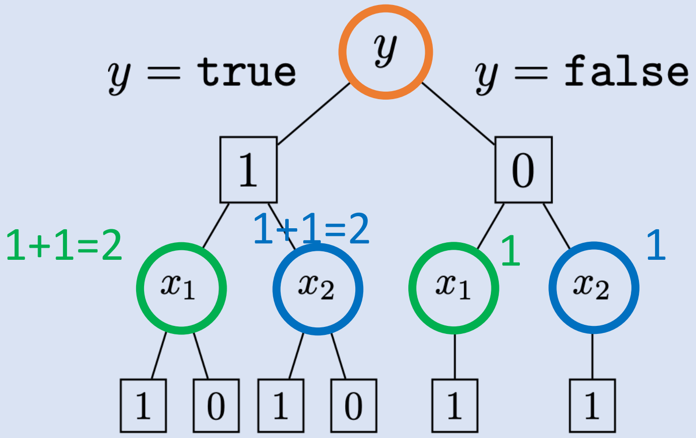

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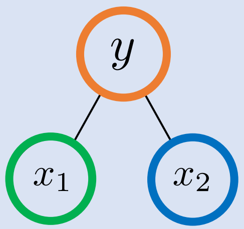

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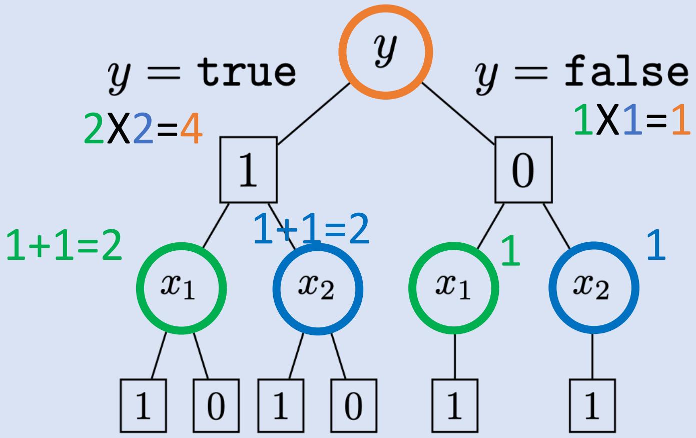

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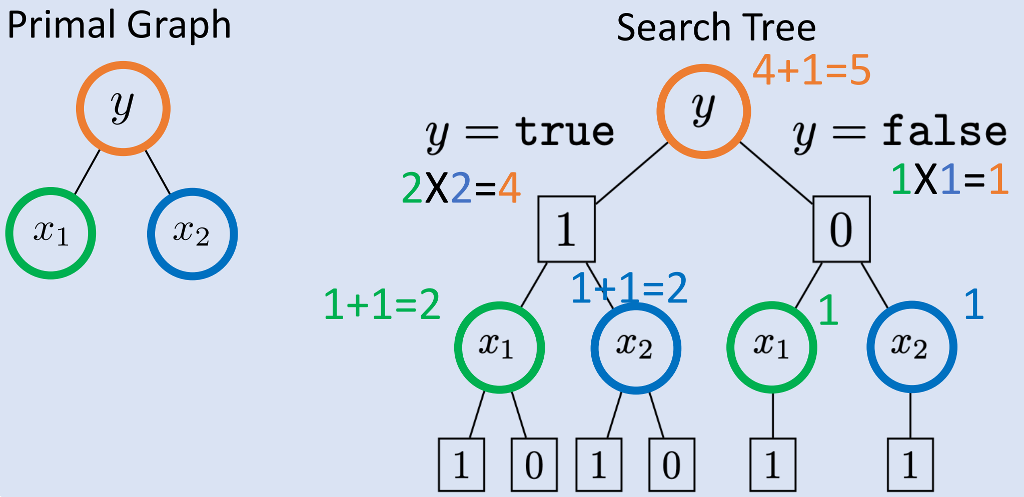

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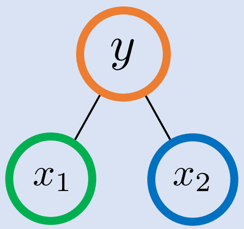

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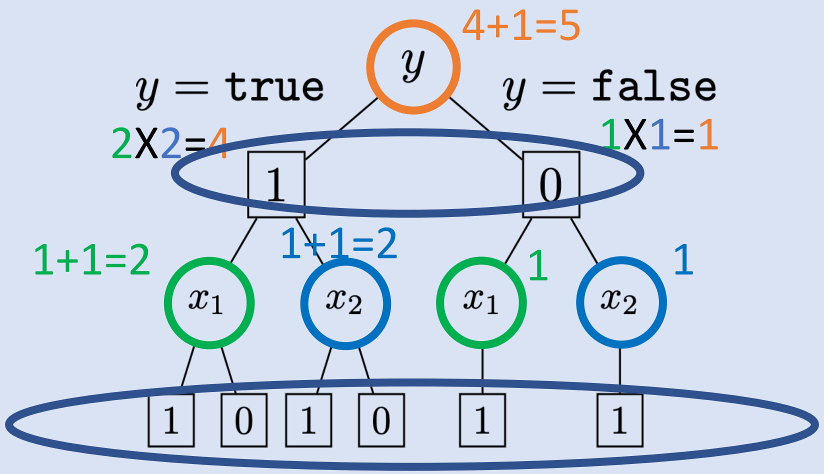
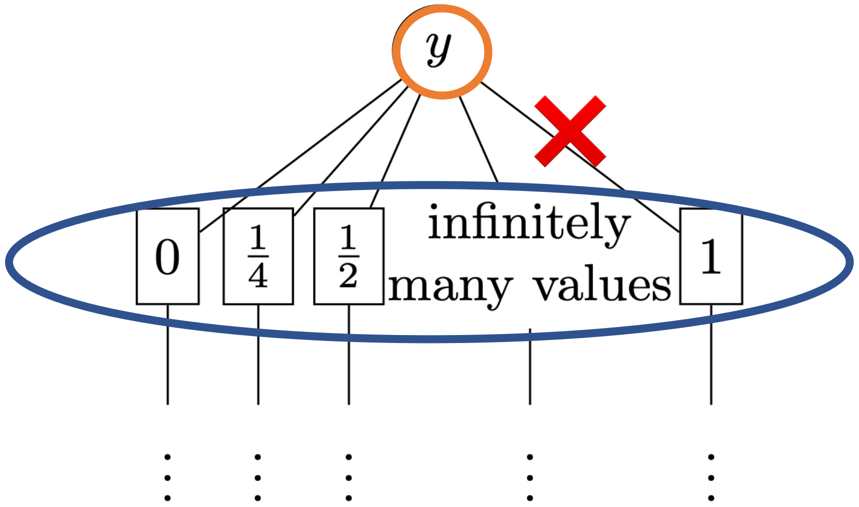


However in hybrid domain …

Instantiate variable with ALL values in its domain



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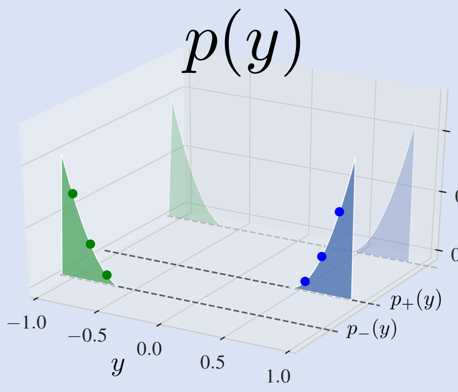
$$MI(\theta) = \int_{-1}^{-0.5} p_-(y)dy + \int_{0.5}^{1} p_+(y)dy$$



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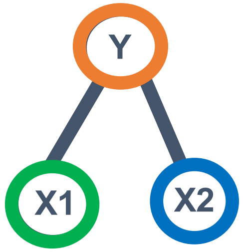

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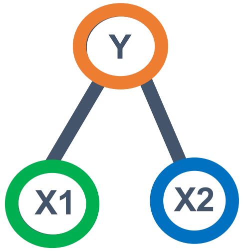

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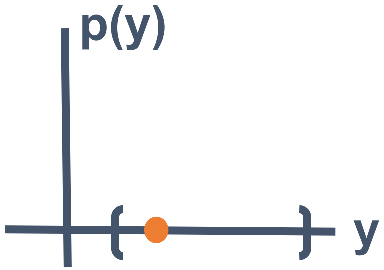

Search-based Model Integration (SMI)

Prop: model integration of SMT($\mathcal{LRA}$) theory is integration over a univariate piecewise polynomial.

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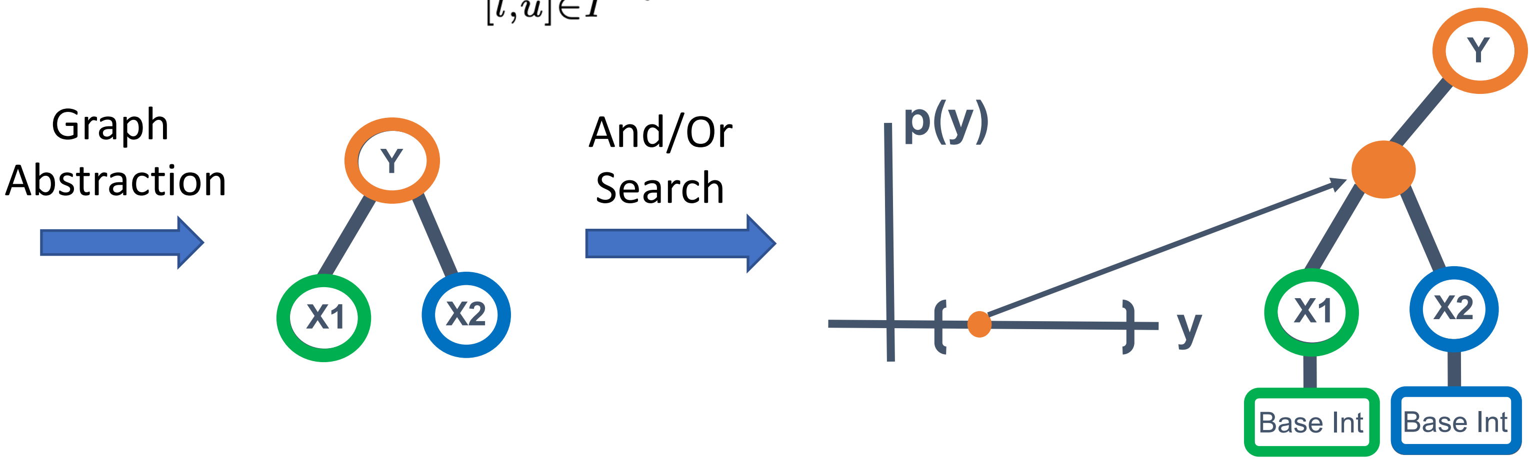

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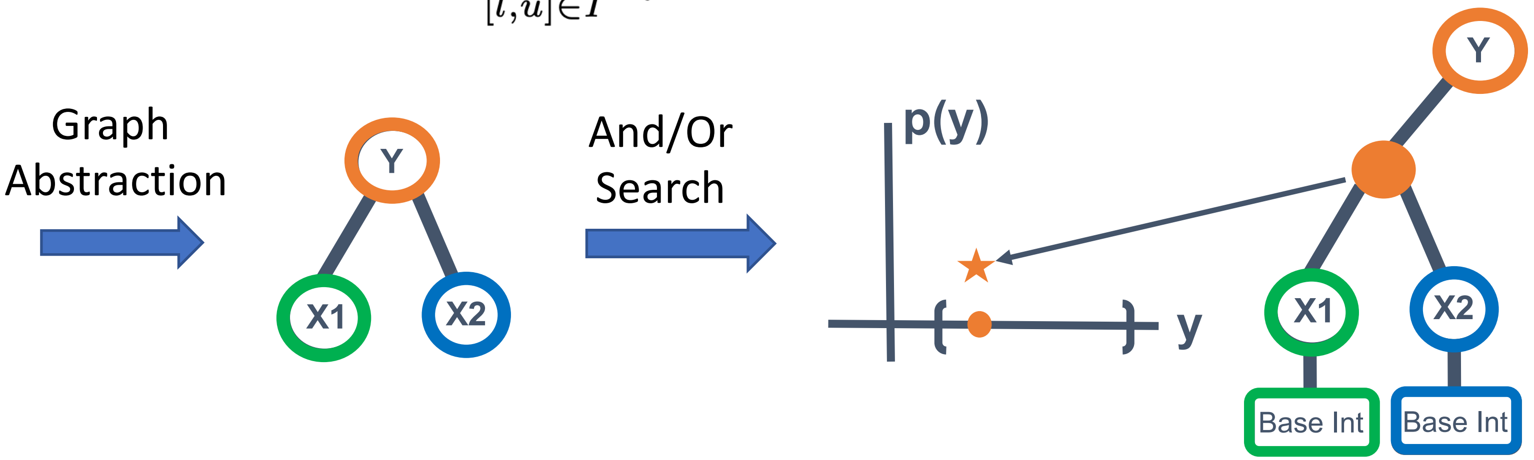

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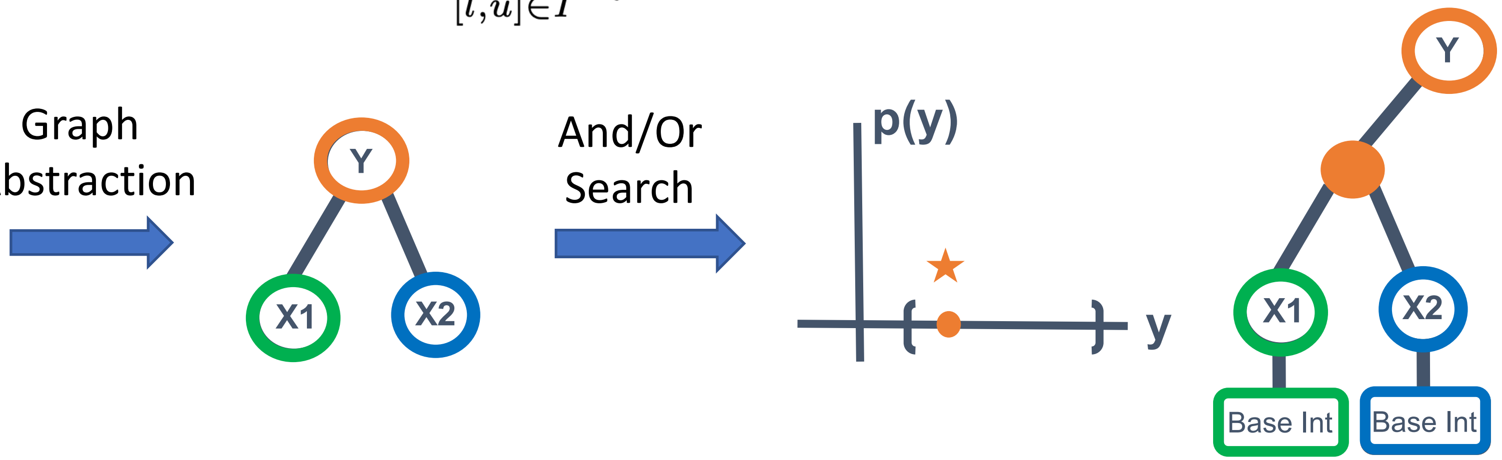

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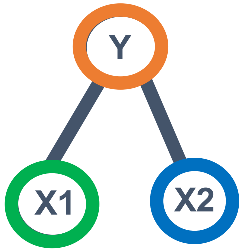

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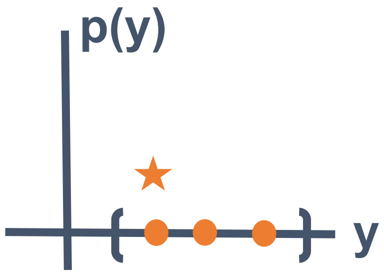

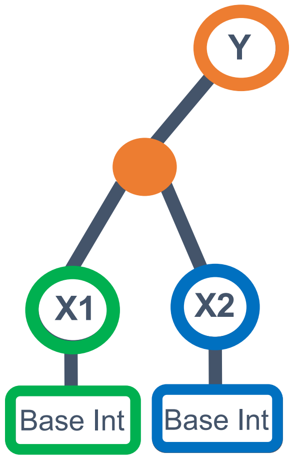

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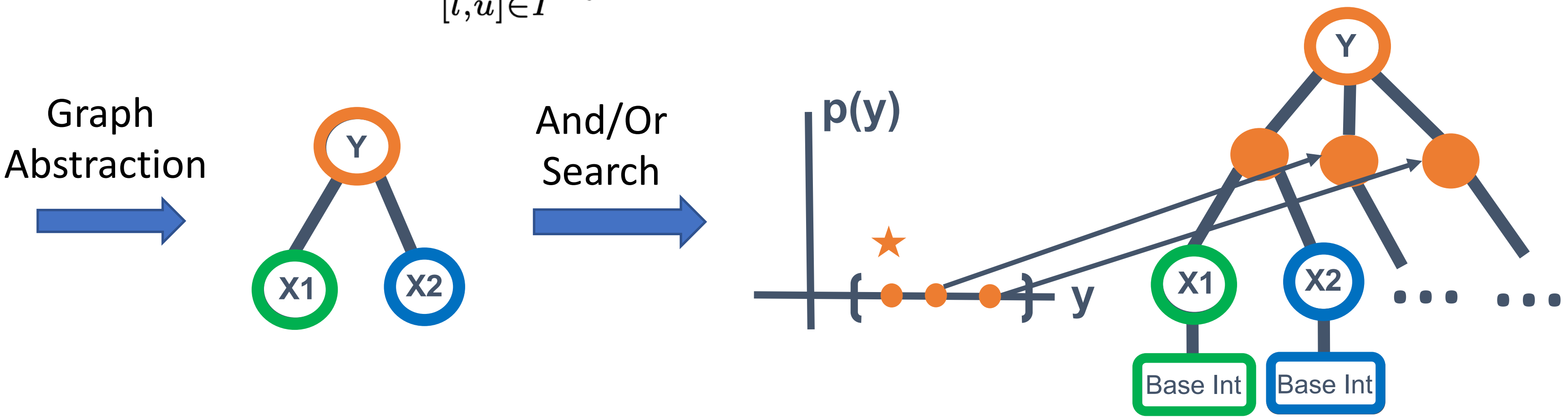

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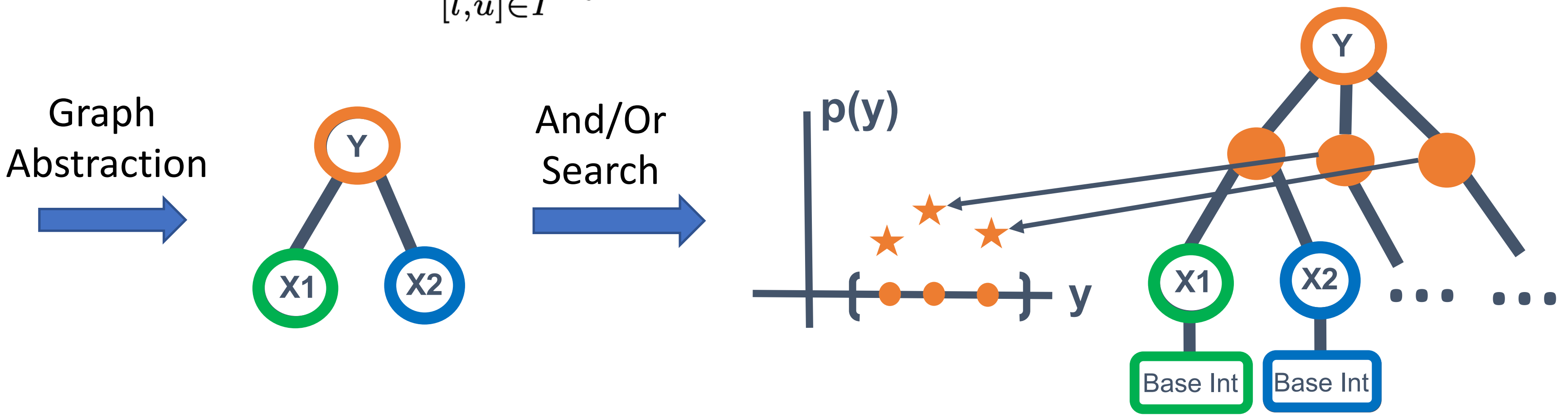

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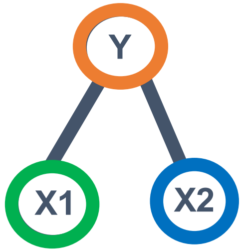

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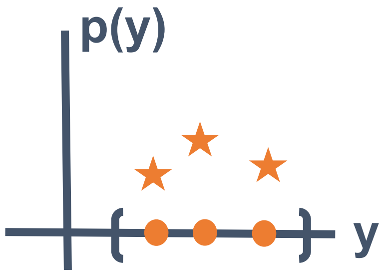

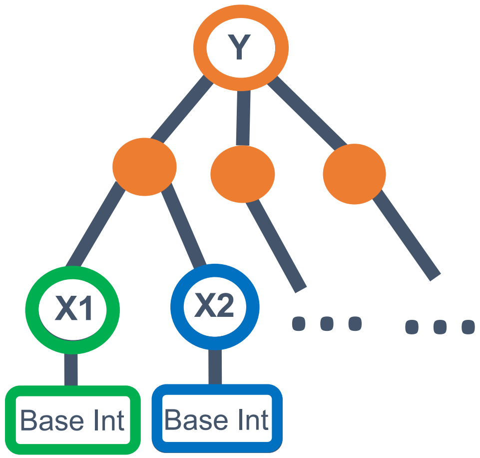

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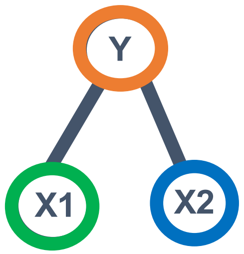

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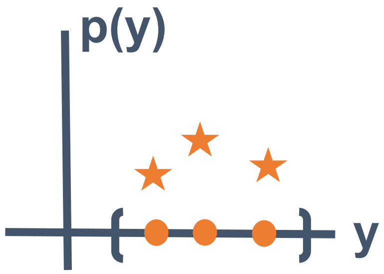
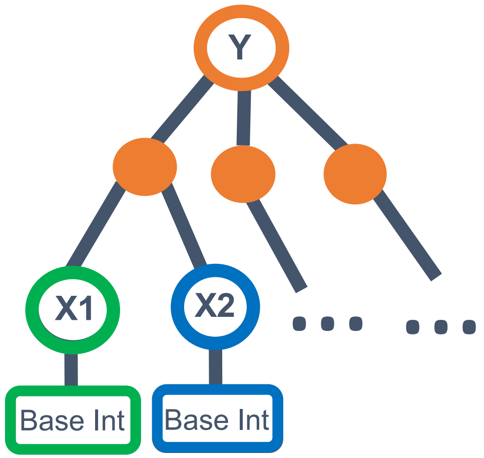
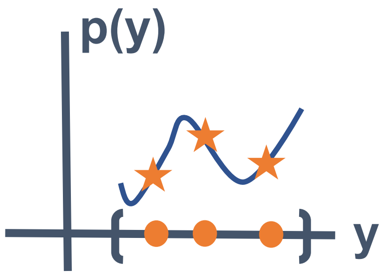




Polynomial Interpolation



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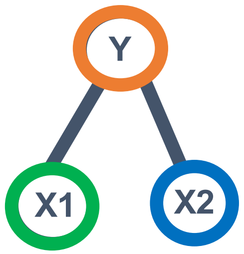

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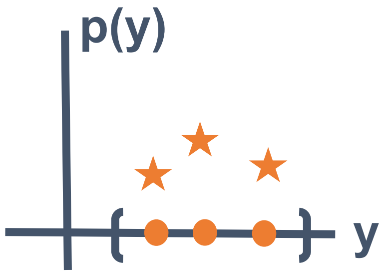

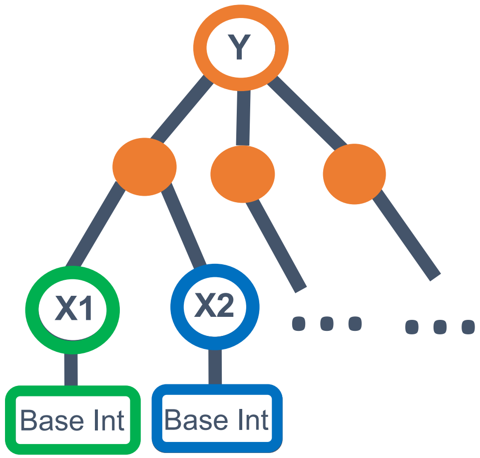

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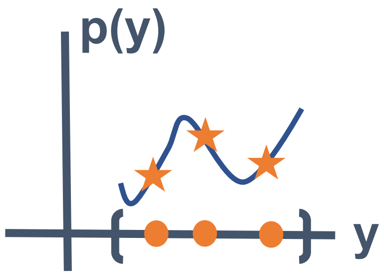


Model Integration

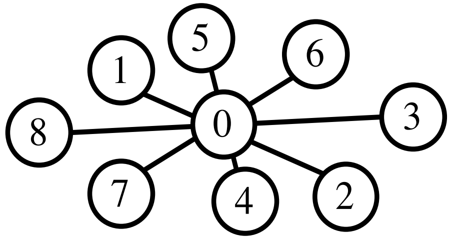
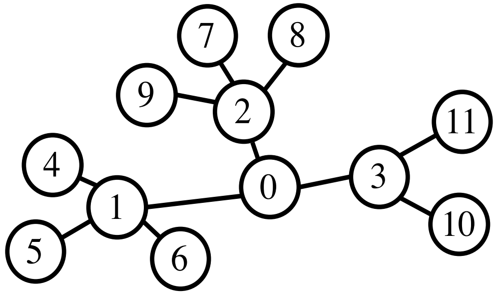
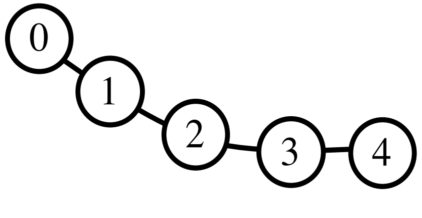
$$MI(\theta) = \sum \int p(y)dy$$

Outline

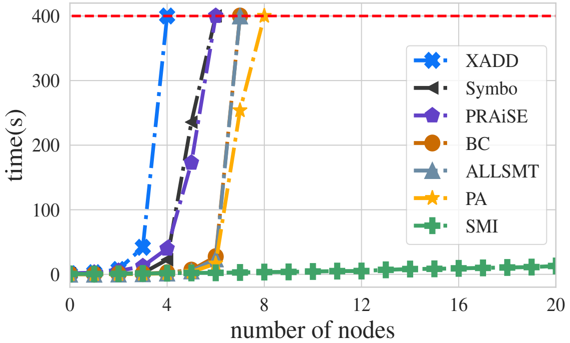
- What is Weighted Model Integration (WMI)?
- What structure to exploit?
- From WMI to Model Integration
- How does Search-Based MI (SMI) work?
- **Complexity & Experimental Results**

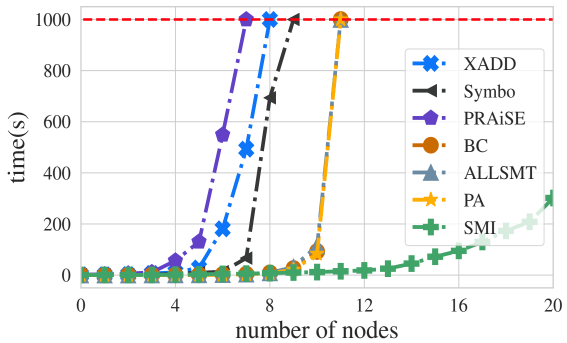
Complexity Analysis

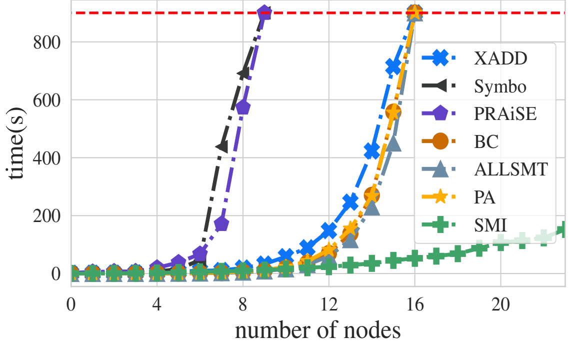
- Search space $O(l \cdot (n^3 \cdot c^{h_p})^{h_t})$ bounded by tree heights h_t, h_p

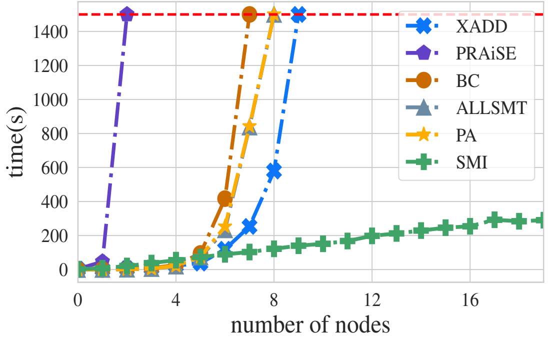
Primal Graph			
Primal Graph h_p	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(n)$
Pseudo Tree h_t	$\mathcal{O}(1)$	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$
Complexity	Polynomial	Quasi-Polynomial	Exponential

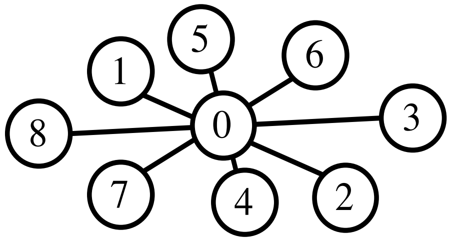
Experimental Results

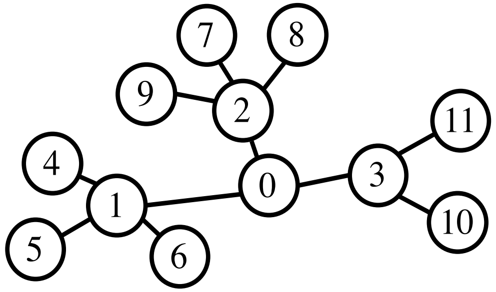


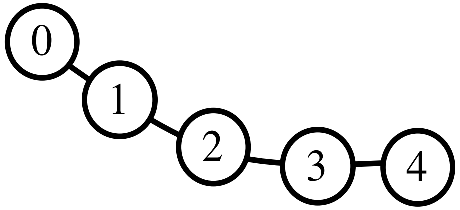


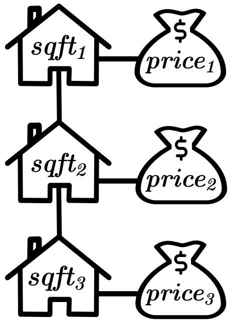












Future Work

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- Approximate WMI solvers, learning weighted SMT($\mathcal{LRA}$) formulas, …

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Thanks for your time!

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