Scaling up Hybrid Probabilistic Inference with Logical and Arithmetic Constraints via Message Passing

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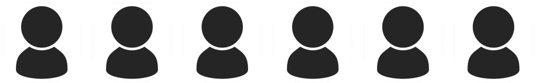
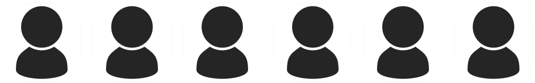
Skill matching system



Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

Skill matching system



- Each ***player*** has a certain skill

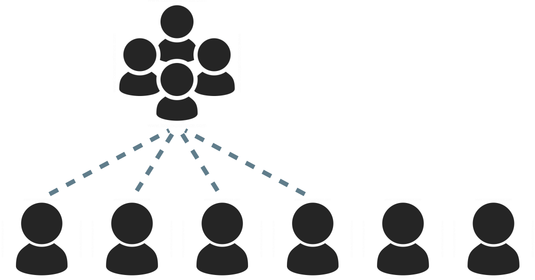
Minka et al., *"Trueskill 2: An improved bayesian skill rating system"*, 2018

Skill matching system

- Each ***player*** has a certain skill
  $\Longrightarrow$ *continuous variables*

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

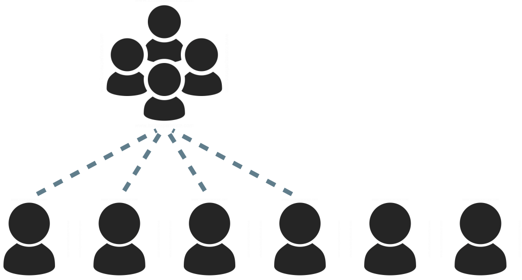
Skill matching system



- Each ***player*** has a certain skill
- Players can form ***teams***

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

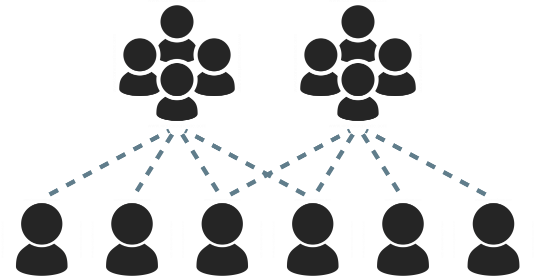
Skill matching system



- Each ***player*** has a certain skill
- Players can form ***teams***
  $\Longrightarrow$ *intricate dependencies*

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

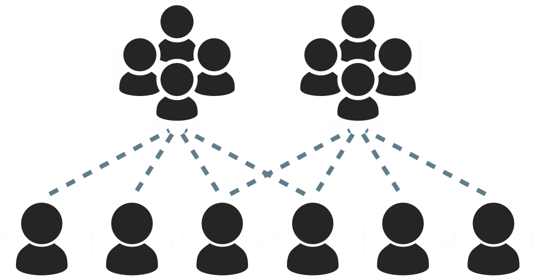
Skill matching system



- Each ***player*** has a certain skill
- Players can form ***teams***
- Each team's skill is bounded by its players' skills

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

Skill matching system

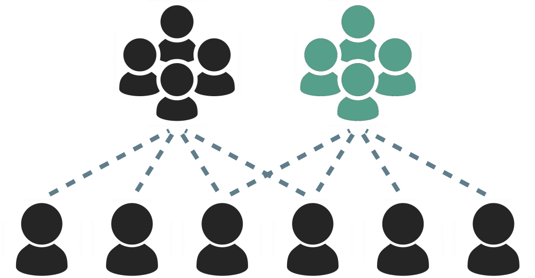


- Each ***player*** has a certain skill
- Players can form ***teams***
- Each team's skill is bounded by its players' skills

$\Rightarrow$ *complex constraints!*

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

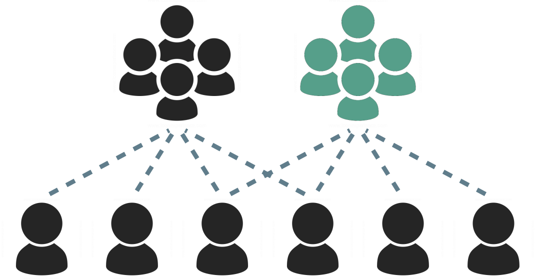
Skill matching system



- Each ***player*** has a certain skill
- Players can form ***teams***
- Each team's skill is bounded by its players' skills
- Good teams form a ***squad***

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

Skill matching system

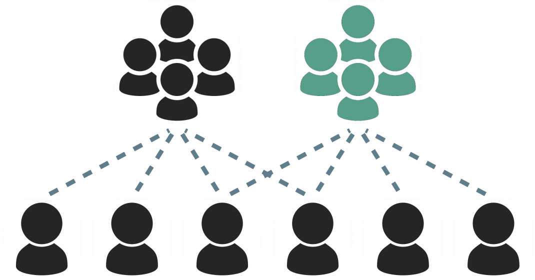


- Each ***player*** has a certain skill
- Players can form ***teams***
- Each team's skill is bounded by its players' skills
- Good teams form a ***squad***

$\Rightarrow$ *discrete variables*

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

Skill matching system



"What is the probability of team T_1 to outperform team T_2, if T_1 is a squad but T_2 is not?"

Minka et al., "Trueskill 2: An improved bayesian skill rating system", 2018

Continuous **+** ***discrete*** **+** ***constraints*** **= ?**

Continuous + ***discrete*** + ***constraints*** = ?

Generative adversarial networks (GANs) *[Goodfellow et al. 2014]*

Variational Autoencoders (VAEs) *[Kingma et al. 2013]*

Continuous + discrete + constraints = ?

~~Generative adversarial networks (GANs)~~ *[Goodfellow et al. 2014]*

~~Variational Autoencoders (VAEs)~~ *[Kingma et al. 2013]*

$\Longrightarrow$ *limited inference capabilities, no constraints*

Continuous + **discrete** + **constraints** = ?

~~Generative adversarial networks (GANs)~~ *[Goodfellow et al. 2014]*

~~Variational Autoencoders (VAEs)~~ *[Kingma et al. 2013]*

Hybrid Bayesian Netowrks (HBNs) *[Heckerman et al. 1995; Shenoy et al. 2011]*

Mixed Probabilistic Graphical Models (MPGMs) *[Yang et al. 2014]*

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$\Longrightarrow$ *strong distributional assumptions*

Continuous + **discrete** + **constraints** = ?

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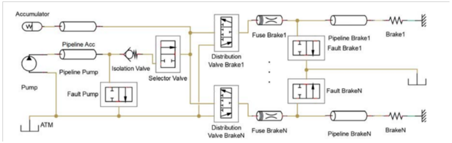
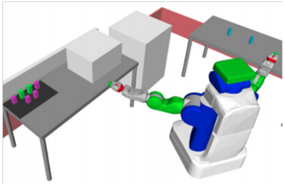
~~Tractable Probabilistic Circuits (PCs)~~ *[Molina et al. 2018; Vergari et al. 2019]*

$\Longrightarrow$ *cannot deal with complex constraints*

Continuous + **discrete** + **constraints** = **SMT**



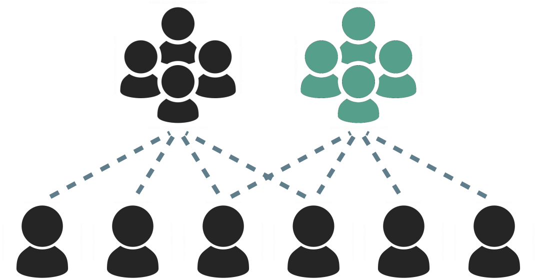


Satisfiability Modulo Theories of the linear arithmetic over the reals (SMT($\mathcal{LRA}$)) delivers all these ingredients by design!

Widely used as a representation language for ***robotics***, ***verification*** and ***planning*** *[Barrett et al. 2010]*

Barrett et al. "Satisfiability modulo theories", 2018

Continuous + **discrete** + **constraints** = **SMT**



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Barrett et al., "Satisfiability modulo theories", 2018

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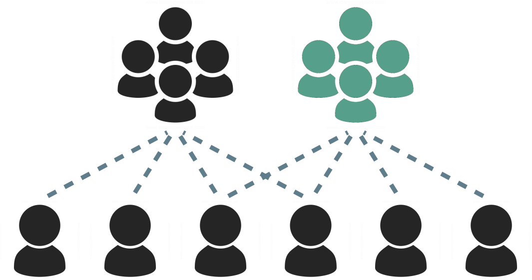
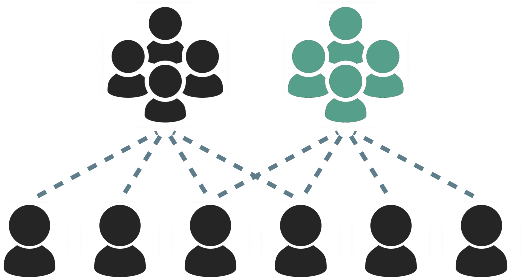


$0 \leq X_{P_i} \leq 10$

for $i = 1, \ldots, N$

Barrett et al., "Satisfiability modulo theories", 2018

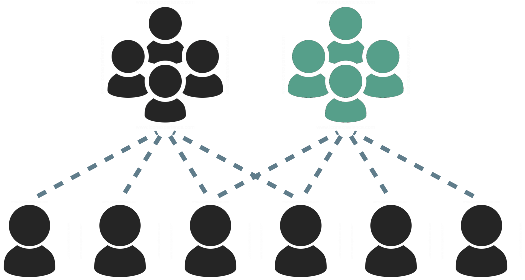
Continuous + discrete + constraints = SMT



- $0 \leq X_{P_i} \leq 10$ for $i = 1, \ldots, N$
- Each team's skill is bounded by its players' skills

Barrett et al., "Satisfiability modulo theories", 2018

Continuous + discrete + constraints = SMT

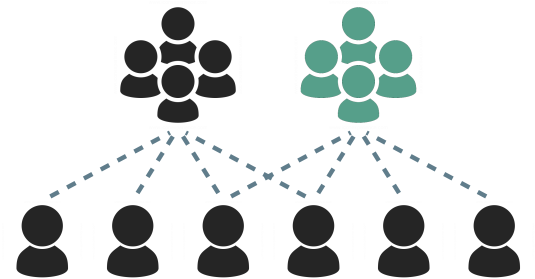


- $0 \leq X_{P_i} \leq 10$
  for $i = 1, \ldots, N$

- $\mid X_{T_j} - X_{P_i} \mid < 1$
  for $j = 1, \ldots, M, i = 1, \ldots, |T_j|$

Barrett et al., "Satisfiability modulo theories", 2018

Continuous + **discrete** + **constraints** = **SMT**



- $0 \leq X_{P_i} \leq 10$
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- $\mid X_{T_j} - X_{P_i} \mid < 1$
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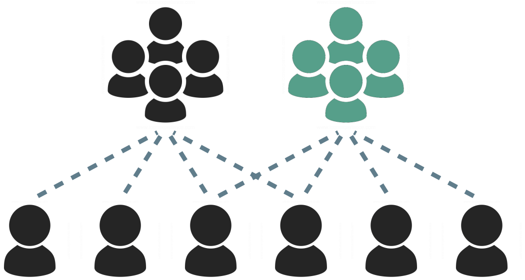
- Good teams form a ***squad***

Barrett et al., "Satisfiability modulo theories", 2018

Continuous + **discrete** + **constraints** = **SMT**



- $0 \leq X_{P_i} \leq 10$
  for $i = 1, \ldots, N$
- $\mid X_{T_j} - X_{P_i} \mid < 1$
  for $j = 1, \ldots, M, i = 1, \ldots, |T_j|$
- $B_{S_j} \Rightarrow X_{T_j} > 2$
  for $j = 1, \ldots, M, i = 1$

Barrett et al., "Satisfiability modulo theories", 2018

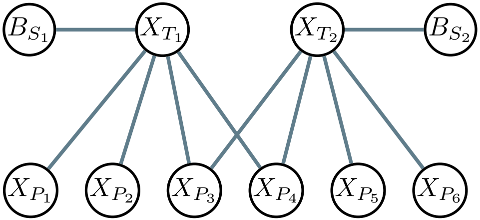
Continuous + discrete + constraints = SMT

$$\Delta = \bigwedge_i 0 \leq X_{P_i} \leq 10 \bigwedge_j \bigwedge_{i \in T_j} \mid X_{T_j} - X_{P_i} \mid < 1 \bigwedge_j (B_{S_j} \Rightarrow X_{T_j} > 2)$$

a single CNF SMT($\mathcal{LRA}$) formula Δ...

Barrett et al., "Satisfiability modulo theories", 2018

Continuous + discrete + constraints = SMT



a single CNF SMT($\mathcal{LRA}$) formula Δ...and its ***primal graph***

Barrett et al., “Satisfiability modulo theories”, 2018

SMT + weights

$$\bigwedge_i 0 \leq X_{P_i} \leq 10$$

$$\bigwedge_j \bigwedge_{i \in T_j} \mid X_{T_j} - X_{P_i} \mid < 1$$

$$\bigwedge_j (B_{S_j} \Rightarrow X_{T_j} > 2)$$

+

$$\begin{cases} w(X_{P_i}), \\ \quad \text{if } 0 \leq X_{P_i} \leq 10 \\ \\ w(X_{T_j}, X_{P_i}), \\ \quad \text{if } \mid X_{T_j} - X_{P_i} \mid < 1 \\ \\ w(B_{S_j}, X_{T_j}), \\ \quad \text{if } B_{S_j} \Rightarrow X_{T_j} > 2 \end{cases}$$

SMT formula Δ

weight functions $\mathcal{W}$

Belle et al., "Probabilistic inference in hybrid domains by weighted model integration", 2015

SMT + **weights** = **Weighted Model Integration**

$$\bigwedge_i 0 \leq X_{P_i} \leq 10$$

$$\bigwedge_j \bigwedge_{i \in T_j} | X_{T_j} - X_{P_i} | < 1$$

$$\bigwedge_j (B_{S_j} \Rightarrow X_{T_j} > 2)$$

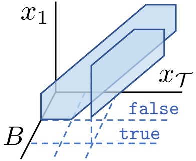
complex support

+

$$\begin{cases} w(X_{P_i}), \\ \quad \text{if } 0 \leq X_{P_i} \leq 10 \\ w(X_{T_j}, X_{P_i}), \\ \quad \text{if } | X_{T_j} - X_{P_i} | < 1 \\ w(B_{S_j}, X_{T_j}), \\ \quad \text{if } B_{S_j} \Rightarrow X_{T_j} > 2 \end{cases}$$

densities

=



(***unnormalized***)

$\text{Pr}_{\Delta}(\mathbf{X}, \mathbf{B})$

Belle et al., "Probabilistic inference in hybrid domains by weighted model integration", 2015

SMT + densities = Weighted Model Integration

Given an SMT($\mathcal{LRA}$) formula Δ over continuous vars $\mathbf{X}$ and discrete ones $\mathbf{B}$, and weight function $\mathcal{W}$, the ***weighted model integral*** (WMI) is

$$\mathsf{WMI}(\Delta, \mathcal{W}; \mathbf{X}, \mathbf{B}) \triangleq \sum_{\boldsymbol{b} \in \mathbb{B}^{|\mathbf{B}|}} \int_{(\mathbf{x}, \boldsymbol{b}) \models \Delta} w(\mathbf{x}, \boldsymbol{b})\, d\mathbf{x}.$$

i.e., computing the ***partition function*** of the unnormalized distribution $\Pr_\Delta$

$\Longrightarrow$ *i.e., integrating the weighted volumes of the feasible regions of* Δ*!*

Belle et al., "Probabilistic inference in hybrid domains by weighted model integration", 2015

WMI

Advanced probabilistic reasoning

"What is the probability of team T_1 to outperform team T_2, if T_1 is a squad but T_2 is not?"

Belle et al., "Probabilistic inference in hybrid domains by weighted model integration", 2015

WMI

Advanced probabilistic reasoning

$$\Phi_S : (B_{S_1} = 1 \wedge B_{S_2} = 0) \implies T_1 \text{ is a squad, } T_2 \text{ is not}$$

$$\Phi_T : (X_{T_1} > X_{T_2}) \implies T_1 \text{ outperforms } T_2$$

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WMI

Advanced probabilistic reasoning

$$\Phi_S : (B_{S_1} = 1 \land B_{S_2} = 0) \implies T_1 \text{ is a squad, } T_2 \text{ is not}$$

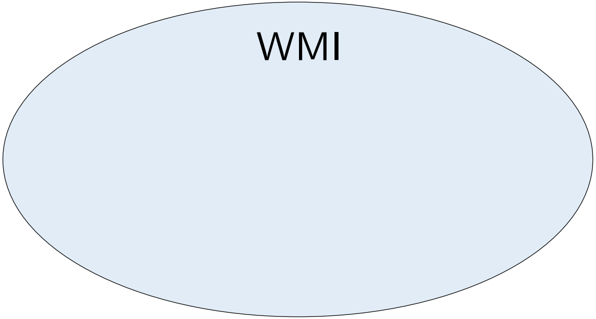
$$\Phi_T : (X_{T_1} > X_{T_2}) \implies T_1 \text{ outperforms } T_2$$

$$\Pr_\Delta(\Phi_T \mid \Phi_S) = \frac{\mathsf{WMI}(\Delta \land \Phi_T \land \Phi_S, \mathcal{W})}{\mathsf{WMI}(\Delta \land \Phi_S, \mathcal{W})} = \frac{4,206}{7,225} \approx 58.22\%$$

$\Rightarrow$ *conditional probabilities as a ratio of two weighted model integrals*

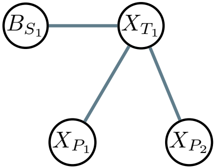
Belle et al., "Probabilistic inference in hybrid domains by weighted model integration", 2015

Tractable WMI



#P-hard in general

treeMI



+

$$\begin{cases} w(X_{P_i}) = X_{P_i} \\ w(X_{T_j}, X_{P_i}) = X_{T_j} X_{P_i} \\ w(B_{S_j}, X_{T_j}) = X_{T_j}^2 \end{cases}$$

=

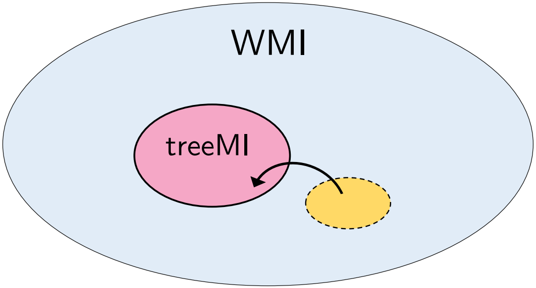
treeMI

[Zeng et al. 2019]

tree-shaped primal graph + ***constrained monomials*** $\mathcal{W}$ = ***polytime WMI inference***

Zeng et al., "Efficient Search-Based Weighted Model Integration", 2019

Tractable WMI

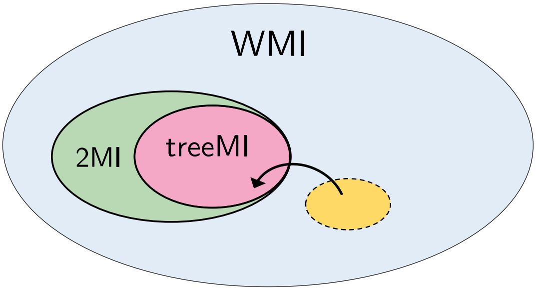


#P-hard in general

largest tractable class known so far

Zeng et al., "Efficient Search-Based Weighted Model Integration", 2019

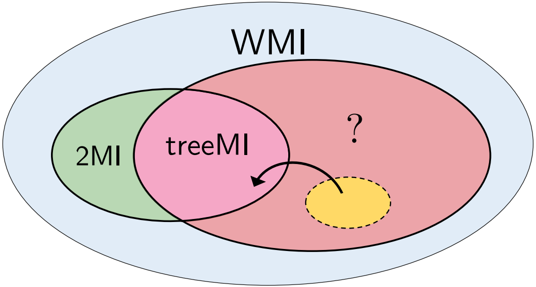
Tractable WMI



- ***#P-hard*** in general
- largest tractable class known so far
- still ***#P-hard***!

Zeng et al., "Efficient Search-Based Weighted Model Integration", 2019

Tractable WMI



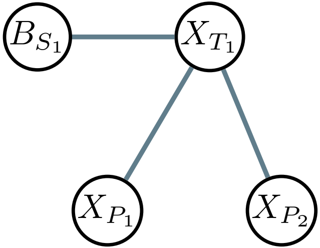
- ***#P-hard*** in general
- largest tractable class known so far
- still ***#P-hard***!
- can we do better?

Zeng et al., "Efficient Search-Based Weighted Model Integration", 2019

MP-WMI

We frame tractable WMI inference at scale as a ***message passing*** scheme...

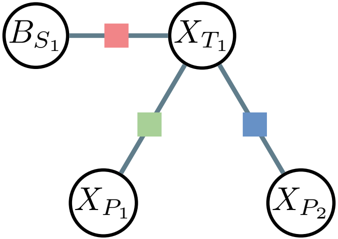
...on primal graphs...



MP-WMI

We frame tractable WMI inference at scale as a ***message passing*** scheme...

...on primal graphs turned into ***factor graphs***



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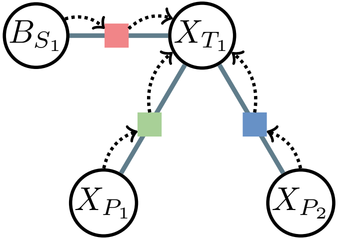
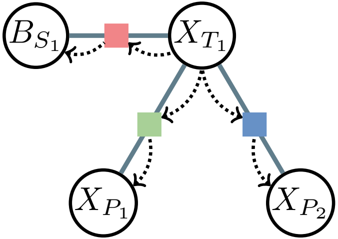


...on primal graphs turned into ***factor graphs***

- comprising an ***upward***

MP-WMI

We frame tractable WMI inference at scale as a ***message passing*** scheme...



...on primal graphs turned into ***factor graphs***

- comprising an ***upward*** and a ***downward*** pass

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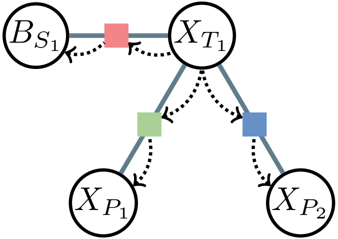


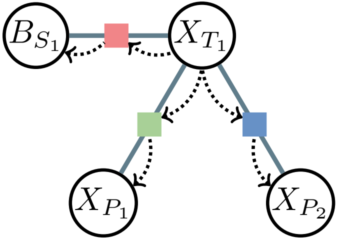
…on primal graphs turned into ***factor graphs***

- comprising an ***upward*** and a ***downward*** pass
- exchanging messages from ***node to factors***

$$\mathsf{m}_{x_i \to f_{\mathcal{S}}}(x_i) = \prod_{f_{\mathcal{S}'} \in \mathsf{neigh}(x_i) \setminus f_{\mathcal{S}}} \mathsf{m}_{f_{\mathcal{S}'} \to x_i}(x_i)$$

MP-WMI

We frame tractable WMI inference at scale as a ***message passing*** scheme...



...on primal graphs turned into ***factor graphs***

- comprising an ***upward*** and a ***downward*** pass
- exchanging messages from ***node to factors***
- and from ***factors to nodes***

$$\mathsf{m}_{f_{ij}\to x_i}(x_i) = \int f_{ij}(x_i, x_j) \cdot \mathsf{m}_{x_j \to f_{ij}}(x_j)\, dx_j$$

Tractable Weight Conditions

Which parametric family Ω for weights to ***guarantee tractable WMI inference***?

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$$\mathsf{m}_{x_i\to f_{\mathcal{S}}}(x_i) = \prod_{f_{\mathcal{S}'}\in\mathsf{neigh}(x_i)\backslash f_{\mathcal{S}}} \mathsf{m}_{f_{\mathcal{S}'}\to x_i}(x_i)$$

Tractable Weight Conditions

Which parametric family Ω for weights to ***guarantee tractable WMI inference***?

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Weights $\mathcal{W} \in \Omega$ should be ***closed under product***...

Tractable Weight Conditions

Which parametric family Ω for weights to ***guarantee tractable WMI inference***?

$$\mathsf{m}_{f_{ij}\to x_i}(x_i) = \int \prod_{\Gamma\in\Delta_{\mathcal{S}}} [\![\mathbf{x}_{\mathcal{S}} \models \Gamma]\!] \prod_{\ell\in\mathcal{L}_\Gamma} w_\ell(\mathbf{x}_{\mathcal{S}})^{[\![\mathbf{x}_{\mathcal{S}}\models\ell]\!]} \cdot \mathsf{m}_{x_j\to f_{ij}}(x_j)\, dx_j$$

$$\mathsf{m}_{x_i\to f_{\mathcal{S}}}(x_i) = \prod_{f_{\mathcal{S}'}\in\mathsf{neigh}(x_i)\setminus f_{\mathcal{S}}} \mathsf{m}_{f_{\mathcal{S}'}\to x_i}(x_i)$$

Weights $\mathcal{W} \in \Omega$ should be ***closed under product***, ***closed under integration***, *and* ***tractable for symbolic integration***

Tractable Weight Conditions

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Weights $\mathcal{W} \in \Omega$ should be ***closed under product***, ***closed under integration***, *and* ***tractable for symbolic integration***

$\Longrightarrow$ *e.g., arbitrary polynomials, exponentiated linear polynomials, etc.*

MP-WMI

An SMT formulation induces a ***piecewise weight representation***

$\Longrightarrow$ *strikingly different from message passing for classical PGMs!*

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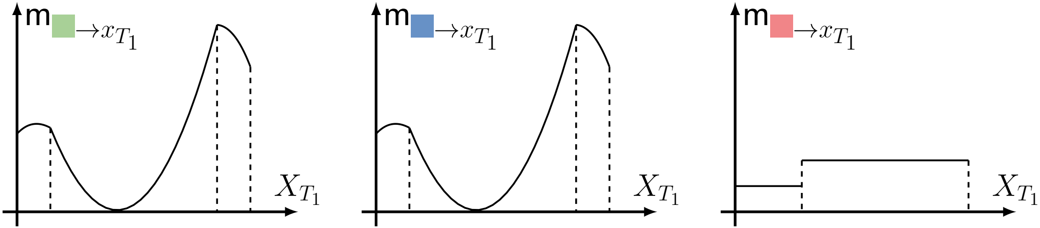

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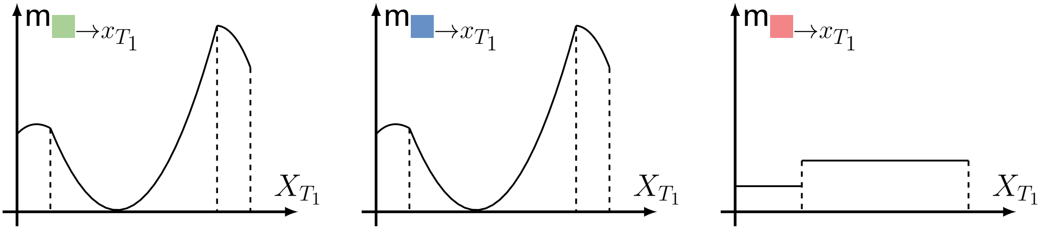
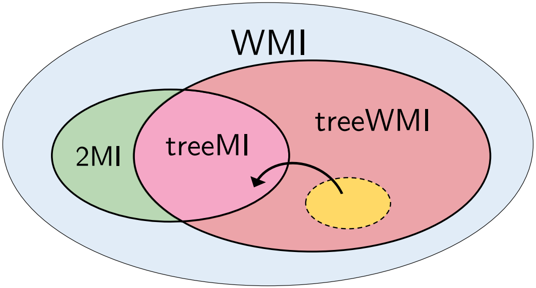


The number of all pieces in MP-WMI is $\mathcal{O}(4nc)^{2d+2}$, where d is the graph diameter

$\Longrightarrow$ *the primal graph should have a* ***bounded diameter!***

Tractable WMI

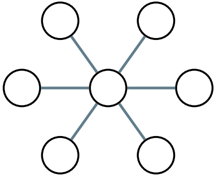


- #P-hard in general
- the largest tractable class known before
- still #P-hard
- ***new largest class***!

Zeng et al., *"Efficient Search-Based Weighted Model Integration"*, 2019

Scaling-up inference

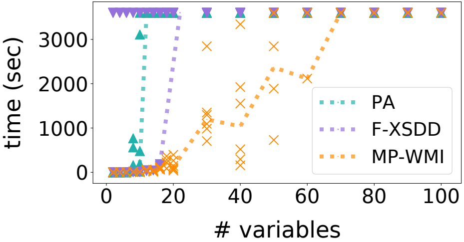
Large set of synthetic benchmarks up to ***N = 100 vars***, 5 trials, ***different primal graphs***



STAR

treewidth: ***1***

diameter: ***2***



MP-WMI takes a ***fraction of the time*** of other exact WMI solvers like PA *[Morettin et al. 2017]* and F-XSDD *[Zuidberg Dos Martires et al. 2019]*

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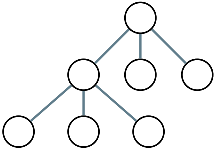
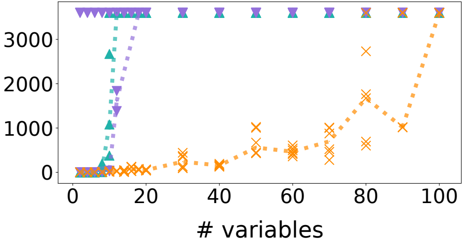


SNOW

treewidth: ***1***

diameter: ***log(N)***



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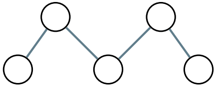
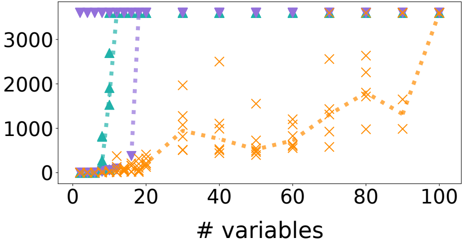


PATH

treewidth: ***1***

diameter: ***N***



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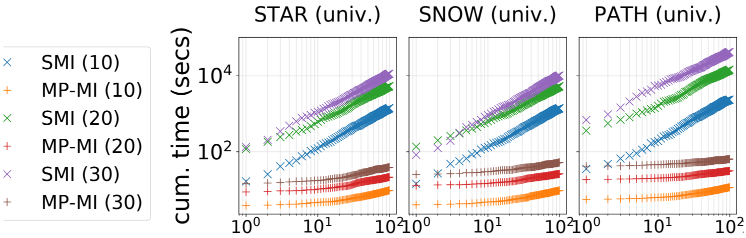
Query amortization

A single message exchange allows to *amortize univariate and bivariate queries*

⟹ *also* ***all marginals*** *and* ***all moments****!*



MP-WMI answers 100 WMI queries faster than competitors solving 10 *[Zeng et al. 2019]*

Conclusions

Real-world data is ***noisy***…

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Real-world data is ***noisy***, ***complex***…

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Real-world data is ***noisy***, ***complex*** and ***mixed continuous-discrete***...

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Code

`github.com/UCLA-StarAI/mpwmi`

References I

- Heckerman, David and Dan Geiger (1995). “Learning Bayesian networks: a unification for discrete and Gaussian domains”. In: *Proceedings of the Eleventh conference on Uncertainty in artificial intelligence*. Morgan Kaufmann Publishers Inc., pp. 274–284.
- Barrett, Clark et al. (2010). “The SMT-LIB initiative and the rise of SMT (HVC 2010 award talk)”. In: *Proceedings of the 6th international conference on Hardware and software: verification and testing*. Springer-Verlag, pp. 3–3.
- Shenoy, Prakash P and James C West (2011). “Inference in hybrid Bayesian networks using mixtures of polynomials”. In: *International Journal of Approximate Reasoning* 52.5, pp. 641–657.
- Kingma, Diederik P and Max Welling (2013). “Auto-encoding variational bayes”. In: *arXiv preprint arXiv:1312.6114*.
- Goodfellow, Ian et al. (2014). “Generative adversarial nets”. In: *Advances in neural information processing systems*, pp. 2672–2680.
- Yang, Eunho et al. (2014). “Mixed graphical models via exponential families”. In: *Artificial Intelligence and Statistics*, pp. 1042–1050.
- Belle, Vaishak, Andrea Passerini, and Guy Van den Broeck (2015). “Probabilistic inference in hybrid domains by weighted model integration”. In: *Proceedings of 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 2770–2776.
- Morettin, Paolo, Andrea Passerini, and Roberto Sebastiani (2017). “Efficient weighted model integration via SMT-based predicate abstraction”. In: *Proceedings of the 26th International Joint Conference on Artificial Intelligence*. AAAI Press, pp. 720–728.
- Barrett, Clark and Cesare Tinelli (2018). “Satisfiability modulo theories”. In: *Handbook of Model Checking*. Springer, pp. 305–343.
- Minka, Tom, Ryan Cleven, and Yordan Zaykov (2018). “Trueskill 2: An improved bayesian skill rating system”. In:

References II

- Molina, Alejandro et al. (2018). “Mixed sum-product networks: A deep architecture for hybrid domains”. In: *Thirty-second AAAI conference on artificial intelligence*.
- Vergari, Antonio et al. (2019). “Automatic Bayesian density analysis”. In: *Proceedings of the AAAI Conference on Artificial Intelligence*. Vol. 33, pp. 5207–5215.
- Zeng, Zhe and Guy Van den Broeck (2019). “Efficient Search-Based Weighted Model Integration”. In: *Proceedings of UAI*.
- Zuidberg Dos Martires, Pedro Miguel, Samuel Kolb, and Luc De Raedt (2019). “How to Exploit Structure while Solving Weighted Model Integration Problems”. In: *UAI 2019 Proceedings*.